IX. Mathematics, Grade 10

Grade 10 Mathematics Test

The spring 2005 Grade 10 MCAS Mathematics Test was based on learning standards in the Massachusetts *Mathematics Curriculum Framework* (2000). The *Framework* identifies five major content strands listed below.

- Number Sense and Operations
- Patterns, Relations, and Algebra
- Geometry
- Measurement
- Data Analysis, Statistics, and Probability

The grade 9–10 learning standards for each of these strands appear on pages 72–75 of the *Mathematics Curriculum Framework*, which is available on the Department Web site at www.doe.mass.edu/frameworks/math/2000/final.pdf.

In *Test Item Analysis Reports* and on the *Subject Area Subscore* pages of the MCAS *School Reports* and *District Reports*, Mathematics test results are reported under five MCAS reporting categories, which are identical to the five *Framework* content strands listed above.

Test Sessions and Content Overview

The MCAS Grade 10 Mathematics Test included two separate test sessions, which were administered on consecutive days. Each session included multiple-choice and open-response questions. Session 1 also included short-answer questions.

Reference Materials and Tools

Each student taking the Grade 10 Mathematics Test was provided with a *Grade 10 Mathematics Reference Sheet*. A copy of the reference sheet follows the final question in this chapter.

During session 2, each student had sole access to a calculator with at least four functions and a square root key. Calculator use was not allowed during session 1. No other reference tools or materials were allowed.

The use of bilingual word-to-word dictionaries was allowed for limited English proficient students only, during both Mathematics test sessions.

Cross-Reference Information

The table at the conclusion of this chapter indicates each item's reporting category and the *Framework* learning standard it assesses. The correct answers for multiple-choice and short-answer questions are also displayed in the table.

Mathematics

SESSION 1

You may use your reference sheet during this session.
*You may **not** use a calculator during this session.*

DIRECTIONS

This session contains fourteen multiple-choice questions, four short-answer questions, and three open-response questions. Mark your answers to these questions in the spaces provided in your Student Answer Booklet.

1 What is the value of the expression below?

$$(3^2 + 3)(3^2 - 3)$$

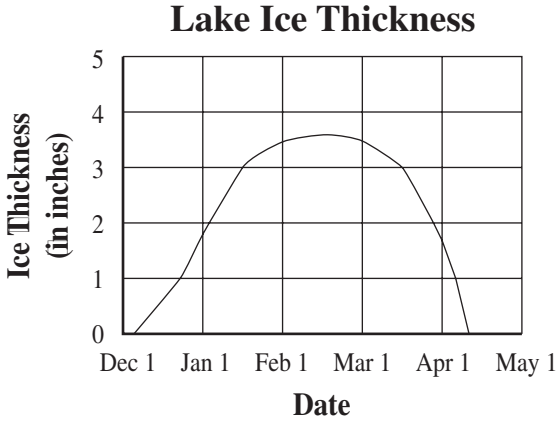
A. 27

B. 72

C. 81

D. 90

2 The graph below shows the thickness of the ice on a lake during the colder months.



Which of the following is closest to the number of days the ice was at least 3 inches thick?

A. 30

B. 45

C. 60

D. 75

3 A fast-growing strain of bacteria doubles in population every 20 minutes. A laboratory has a culture of 200 of these bacteria cells. The function below can be used to find p, the number of bacteria cells in this culture after t **hours.**

$$p = 200(8^t)$$

Which of the following is closest to the total number of bacteria cells after 2 hours?

A. 3,200

B. 12,800

C. 51,200

D. 2,560,000

4 What is the value of the expression below?

$$\left|2^3 - 3^2\right|$$

A. 0

B. 1

C. 2

D. 3

5 Which of the following is a factor of the polynomial below?

$$4x^3y - 8x^2y^2 + 10xy^3$$

A. $4y^2$

B. $2x^2$

C. $2xy$

D. x^2y^2

6 The Golden Ratio is defined by the expression shown below.

$$\frac{1 + \sqrt{5}}{2}$$

Which of the following is closest to the value of the ratio?

A. 1.1

B. 1.6

C. 2.1

D. 2.9

7 In which equation below is the solution equal to the multiplicative inverse of $\frac{2}{3}$?

A. $\frac{2}{3} \cdot r = 1$

B. $\frac{2}{3} \cdot r = \frac{2}{3}$

C. $\frac{2}{3} \cdot r = 0$

D. $\frac{2}{3} \cdot r = -1$

8 Which of the following is closest to $\frac{2^5 \cdot 5^3}{33}$?

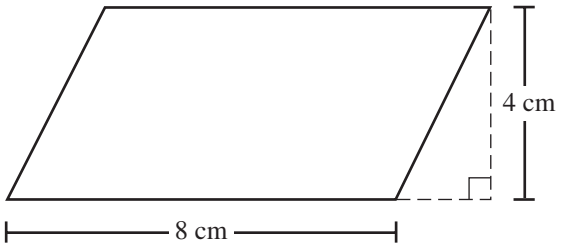
A. 5

B. 12

C. 120

D. 1200

9 What is the area of the parallelogram represented below?



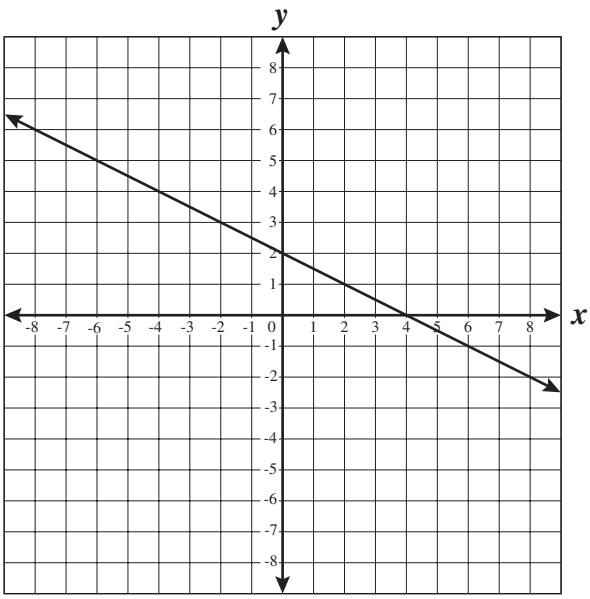
A. 32 cm^2

B. 24 cm^2

C. 16 cm^2

D. 12 cm^2

10 Which of the following best represents the equation of the line shown on the graph below?



A. $y = -\frac{1}{2}x + 2$

B. $y = -2x + 2$

C. $y = -\frac{1}{2}x + 4$

D. $y = -2x + 4$

11 At a fish market, Mr. Estes bought several pounds of cod that was on sale for $3.59 per pound. The total cost of the cod that he bought was $28.63.

Which of the following is closest to the amount of cod that Mr. Estes bought?

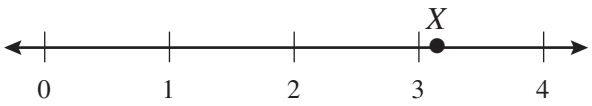
A. 6 pounds

B. 7 pounds

C. 8 pounds

D. 9 pounds

12 Point X is graphed on the number line as shown below.



Which of the following numbers is closest to the location of point X?

A. $\sqrt{6}$

B. $\sqrt{8}$

C. $\sqrt{11}$

D. $\sqrt{13}$

13 If the denominator is not zero, which of the following is equivalent to the expression below?

$$\frac{6x^3 - 12x^2 - 9x}{3x}$$

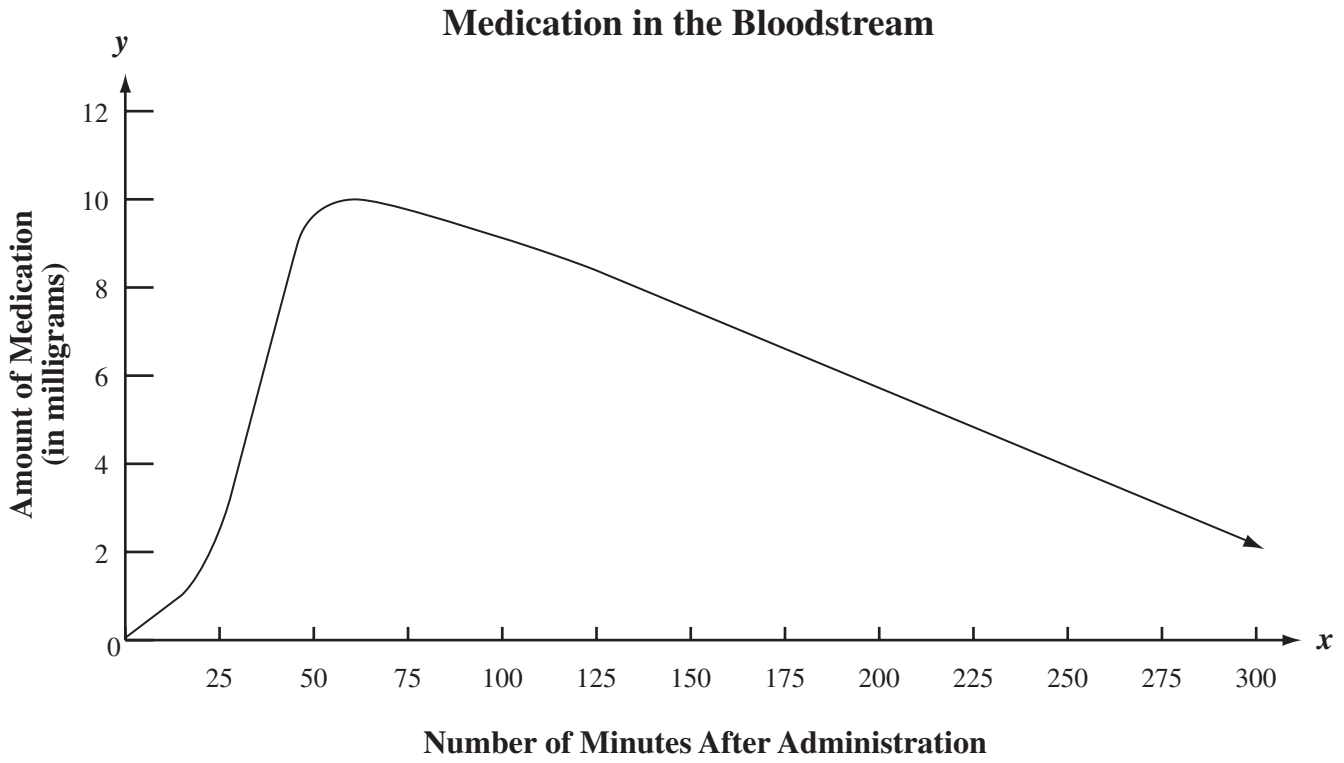
A. $6x^3 - 12x^2 - 3$

B. $2x^2 - 12x^2 - 9x$

C. $6x^2 - 4x - 3$

D. $2x^2 - 4x - 3$

14 The graph below shows the number of milligrams of a medication in the bloodstream from the time it was administered to 300 minutes after administration.



Using the information from the graph, which of the following statements is true?

A. The maximum amount of medication in the bloodstream was 12 milligrams.

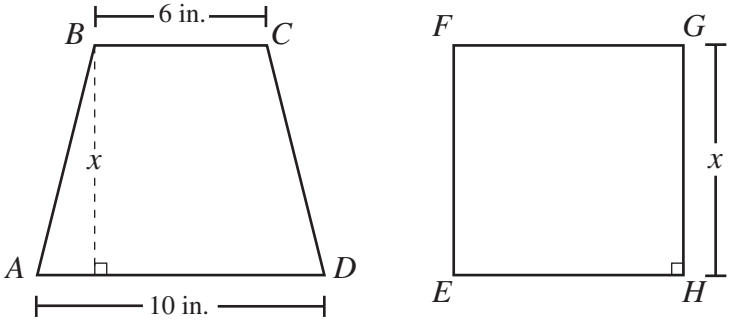
B. The minimum amount of medication was in the bloodstream 300 minutes after administration.

C. The amount of medication in the bloodstream increased at a faster rate than it decreased.

D. The maximum amount of medication was in the bloodstream 100 minutes after administration.

Questions 15 and 16 are short-answer questions. Write your answers to these questions in the boxes provided in your Student Answer Booklet. Do not write your answers in this test booklet. You may do your figuring in the test booklet.

15 Trapezoid *ABCD* shown below has bases measuring 6 inches and 10 inches and a height of x inches. Square *EFGH* shown below has sides measuring x inches. The trapezoid and the square have equal areas.



What is the value of x, in inches?

16 On an airline, approximately 10% of the airline passengers who are booked for a flight do not show up for the flight. The airline has booked 160 passengers for a flight with maximum seating of 135. How many of the 160 passengers booked for this flight will **not** have a seat, assuming 10% of the booked passengers do not show up?

Question 17 is an open-response question.

- **BE SURE TO ANSWER AND LABEL ALL PARTS OF THE QUESTION.**
- **Show all your work (diagrams, tables, or computations) in your Student Answer Booklet.**
- **If you do the work in your head, explain in writing how you did the work.**

Write your answer to question 17 in the space provided in your Student Answer Booklet.

17 Quinn works in Chicago and in New York City. He travels by taxi in each of the two cities.

In Chicago, he pays a fixed taxi fare of $1.90 per ride, plus $1.60 per mile traveled.

a. Write an equation that expresses f, Quinn's total fare for a taxi ride in Chicago, as a function of m, the number of miles traveled.

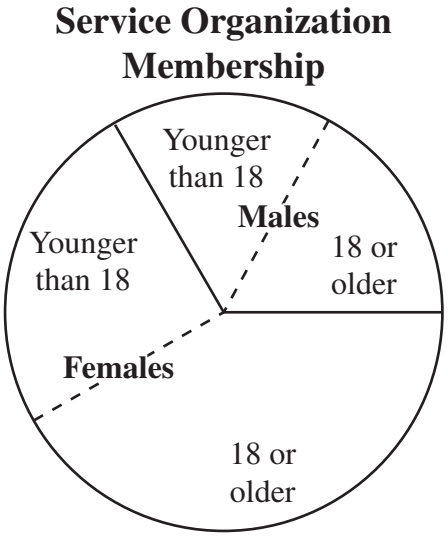
In New York City, Quinn pays a fixed taxi fare of $1.50 per ride, plus 25¢ per $\frac{1}{10}$ mile traveled.

b. Write an equation that expresses f, Quinn's total fare for a taxi ride in New York City, as a function of m, the number of **miles** traveled.

c. On a recent trip Quinn noticed that the total number of miles traveled by taxi from the airport to the hotel was the same in each of the two cities. Before tips were added, his taxi fare to the hotel in New York City was $12.20 more than his taxi fare to the hotel in Chicago. What was the distance from the airport to the hotel in each city? Show or explain how you got your answer.

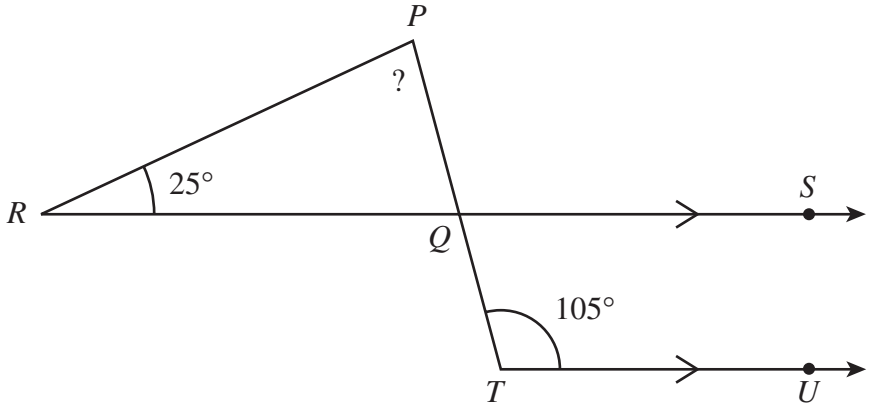
Questions 18 and 19 are short-answer questions. Write your answers to these questions in the boxes provided in your Student Answer Booklet. Do not write your answers in this test booklet. You may do your figuring in the test booklet.

18 The circle graph shown below represents the membership of a service organization. In this organization, $\frac{2}{3}$ of the members are female.



Approximately what fractional part of the total membership consists of males who are 18 or older?

19 In the figure shown below, $\overrightarrow{RS}$ is parallel to $\overrightarrow{TU}$, and $\overline{PT}$ intersects $\overrightarrow{RS}$ at Q.



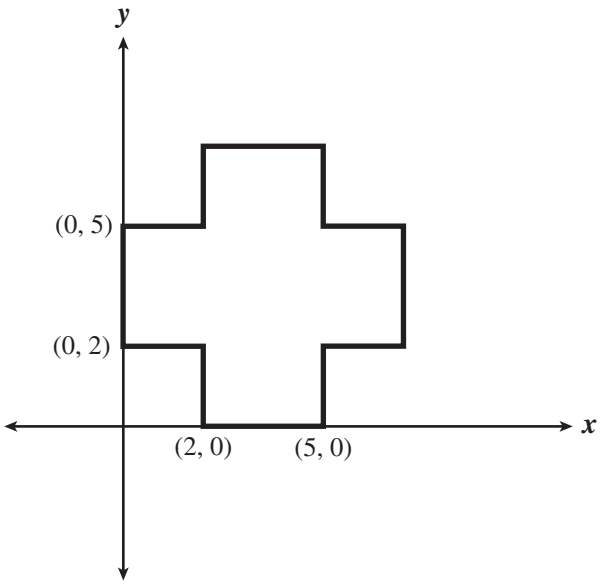
What is the measure of $\angle RPQ$?

Questions 20 and 21 are open-response questions.

- **BE SURE TO ANSWER AND LABEL ALL PARTS OF EACH QUESTION.**
- **Show all your work (diagrams, tables, or computations) in your Student Answer Booklet.**
- **If you do the work in your head, explain in writing how you did the work.**

Write your answer to question 20 in the space provided in your Student Answer Booklet.

20 A landscaper's scale model of a patio is shown on the coordinate plane below. All of the corners of the patio form right angles, and the patio has both a horizontal and a vertical line of symmetry. The coordinates of some of the vertices are shown.



Scale
1 unit represents 1 yard

a. What is the total perimeter of the patio, in yards? Show or explain how you got your answer.

b. What is the total area of the patio, in square yards? Show or explain how you got your answer.

c. A circular fountain will be placed in the center of the patio at the point where the patio's lines of symmetry intersect. What are the coordinates of the point that shows where the center of the fountain will be placed? Show or explain how you got your answer.

Write your answer to question 21 in the space provided in your Student Answer Booklet.

21 Carla can use 100 square feet of floor space in her school's gymnasium, in any way she chooses, to set up computer stations for a science fair. She has chosen to use floor space in the shape of a rectangle, with dimensions that are **whole numbers**.

a. Draw all possible rectangles with an area of 100 square feet and whole-number dimensions. Your drawings do not have to be to scale, but you must label the dimensions on each drawing.

b. Carla plans to buy a length of rope to surround her floor space. Which rectangle that you drew in part a. has the smallest perimeter and will thus require the least amount of rope? Show or explain how you got your answer.

c. To set up her computer stations, Carla will subdivide her rectangular floor space into small rectangles that each measure 2 feet by 4 feet.

- Using the rectangle you chose in part b. as Carla's floor space, what is the maximum number of these small rectangles that she can create?
- To support your answer, sketch the rectangle from part b. subdivided into the maximum number of these small rectangles.
- Explain how you know your answer is correct.

Mathematics

SESSION 2

You may use your reference sheet during this session.
You may use a calculator during this session.

DIRECTIONS

This session contains eighteen multiple-choice questions and three open-response questions. Mark your answers to these questions in the spaces provided in your Student Answer Booklet.

22 Four major underwater tunnels were constructed in New York City between 1925 and 1950. The tunnels and their lengths are listed in the chart below.

New York City Tunnel Lengths

Tunnel Name	Length (kilometers)	Year Completed
Holland	2.6	1927
Lincoln	2.5	1937
Queens-Midtown	1.9	1940
Brooklyn-Battery	2.8	1950

Which of the following is closest to the mean of these four lengths?

A. 2.20 kilometers

B. 2.35 kilometers

C. 2.45 kilometers

D. 2.55 kilometers

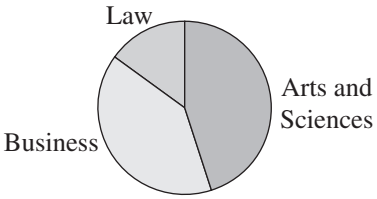
23 A local university is divided into three colleges. The table below shows the number of students enrolled in each college.

University Enrollment

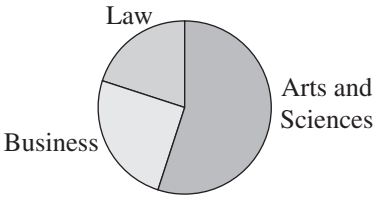
College	Number of Students
Arts and Sciences	8036
Business	2977
Law	1014

Which of the following circle graphs best represents the data in the table?

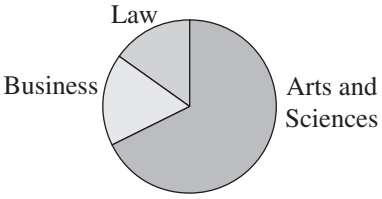
A. **University Enrollment**



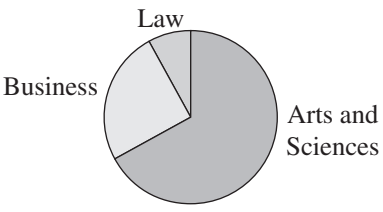
B. **University Enrollment**



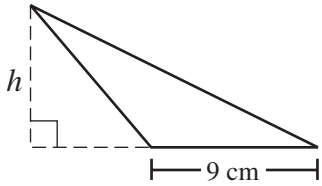
C. **University Enrollment**



D. **University Enrollment**



24 What is h, the height of the triangle represented below, if its area is 58.5 square centimeters?



A. 13 cm

B. 18 cm

C. 26 cm

D. 39 cm

25 $\overline{AB}$ has one endpoint at $A(2, 5)$, and its midpoint is at (4, 0). What are the coordinates of B, the other endpoint of $\overline{AB}$?

A. (2, −5)

B. (3, 2.5)

C. (6, −5)

D. (6, 2.5)

26 Of the people in attendance at a recent baseball game,

- one-third had grandstand tickets,
- one-fourth had bleacher tickets, and
- the remaining 11,250 people in attendance had other tickets.

What was the total number of people in attendance at the game?

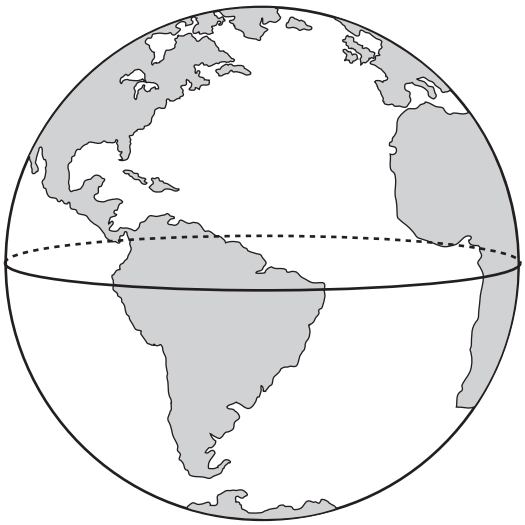
A. 27,000

B. 20,000

C. 16,000

D. 18,000

27 Tiffany wants to calculate the volume of her globe. The globe is in the shape of a sphere, as represented by the picture below. She measured the circumference of the globe along the equator to be 24 inches.



Which of the following measures is closest to the volume of Tiffany's globe?

A. 46 cubic inches

B. 61 cubic inches

C. 183 cubic inches

D. 234 cubic inches

28 Marcella's homeroom had a party at a local arcade. Each of the 26 students attending played the same game. Marcella recorded the number of points that each student scored for that game and put the data into score intervals. The results are shown in the chart below.

Arcade Game Scores

Score Interval (in points)	Number of Students
0 through 100	4
101 through 200	6
201 through 300	4
301 through 400	7
401 through 500	3
501 through 600	2

Based on the information in the chart, which interval contains the median score?

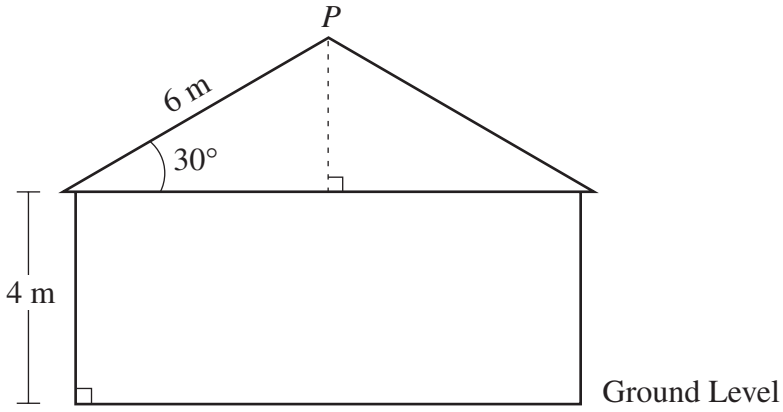
A. 101 through 200

B. 201 through 300

C. 301 through 400

D. 401 through 500

29 The diagram below shows the side view of a house. The base of its roof is 4 meters above ground level.



Point *P* is the highest point on the roof. Based on the diagram, what is the distance from *P* to ground level?

A. 6 m

B. 7 m

C. 10 m

D. 13 m

30 The stem-and-leaf plot below shows the scores on a history exam.

Exam Scores

5	8
6	2 7
7	0 3 3 6
8	4 5 5
9	2 2 2 4 7
10	0

Key
6 | 2 represents 62

Which of the following measures of the data is greatest?

A. mean

B. median

C. mode

D. range

Question 31 is an open-response question.

- **BE SURE TO ANSWER AND LABEL ALL PARTS OF THE QUESTION.**
- **Show all your work (diagrams, tables, or computations) in your Student Answer Booklet.**
- **If you do the work in your head, explain in writing how you did the work.**

Write your answer to question 31 in the space provided in your Student Answer Booklet.

31 A designer at Royal Jewelers wants to create a 10-ounce necklace that will be made of gold and silver. The necklace will have a total value of $206.50.

a. Write an equation that represents the total weight of the 10-ounce necklace if it contains g ounces of gold and s ounces of silver.

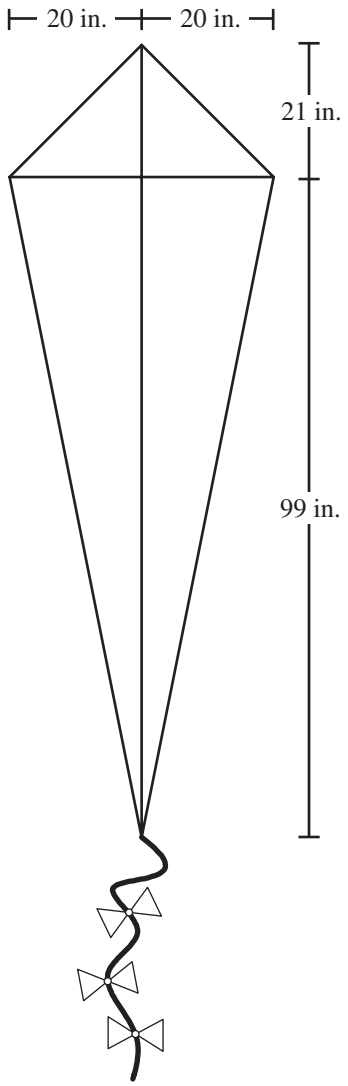
b. Given that the value of gold is $318 per ounce and the value of silver is $5 per ounce, write an equation in terms of g and s that represents the total value of the 10-ounce necklace.

c. The two equations from parts a. and b. form a system. Solve the system of equations for g and s. Show all of your work.

d. What will be the value, in dollars, of the gold in the 10-ounce necklace? Show or explain how you got your answer.

Mark your answers to multiple-choice questions 32 through 40 in the spaces provided in your Student Answer Booklet. Do not write your answers in this test booklet. You may do your figuring in the test booklet.

32 A kite has perpendicular diagonals with the measures shown in the drawing below.



What is the perimeter, in inches, of the kite?

A. 130

B. 165

C. 260

D. 310

33 Deborah decided to mow lawns to earn the $280 she needs for a school orchestra trip. If she earns $18 per lawn, what is the **minimum** number of lawns she needs to mow to earn the money for the trip?

A. 15

B. 16

C. 18

D. 20

34 Which of the following is equivalent to the expression below?

$$(x - 2)(2x^2 + 3) + x^3 - 2x$$

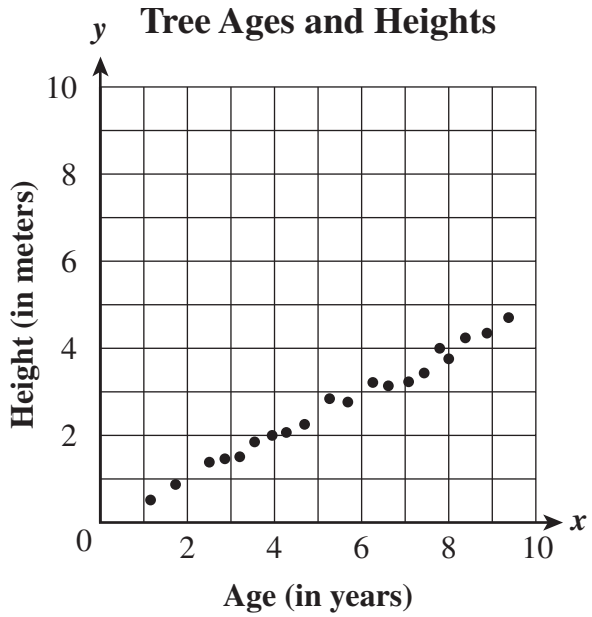
A. $3x^3 - 2x - 6$

B. $3x^3 + x - 6$

C. $3x^3 - x^2 - 2x - 6$

D. $3x^3 - 4x^2 + x - 6$

35 The scatterplot below shows the ages and heights of 20 trees on a tree farm.



If x = age in years and y = height in meters, which of the following equations best approximates the line of best fit for this scatterplot?

A. $y = \frac{1}{2}x$

B. $y = \frac{1}{2}x + 5$

C. $y = 2x$

D. $y = 2x + 5$

36 The table below shows the test scores of 7 students. The scores are in order from least to greatest.

Test Scores

Student	Score
Janet	72
Mark	75
Luisa	77
Byron	81
Ray	84
Devin	86
Kamara	90

Which of the following would change the median of the scores?

A. adding 5 points to Janet's score

B. adding 5 points to Devin's score

C. subtracting 5 points from Ray's score

D. subtracting 5 points from Luisa's score

37 The chart below separates the number of students majoring in math/science from students pursuing other majors at a state college.

Students' Majors by Class

	Freshmen	Sophomores	Juniors	Seniors
Math/Science Majors	260	310	200	330
Other Majors	1390	1510	1450	1550

What percent of the math/science majors are seniors?

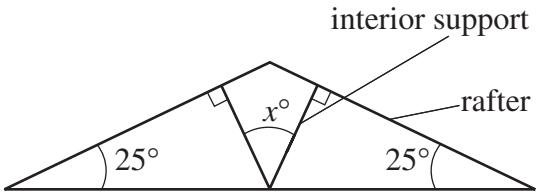
A. 43%

B. 30%

C. 21%

D. 5%

38 Each of the two interior supports for part of a roof is perpendicular to a rafter, as shown below.



What is x, the measure, in degrees, of the angle formed by the two interior supports?

A. 50

B. 65

C. 90

D. 130

39 Kelly wants to buy a tool set that is on sale at a hardware store. The price of each tool set will be decreased by 8% each morning just before the store opens. The sale will last for 7 days, or until all the sets are sold.

After the first reduction on Monday, the price of each set was $135.

If Kelly wants to wait until the first day that the price is $100 or less, on which day should she buy her tool set, if one is still available?

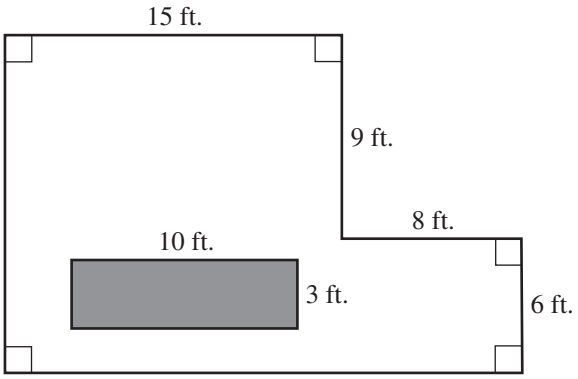
A. Wednesday

B. Thursday

C. Friday

D. Saturday

40 How many square feet of carpeting are needed to cover the floor of the room represented by the drawing below? Note that the shaded region is to be left uncovered to leave space for the construction of a built-in trophy case with a rectangular base.



A. 125 sq. ft.

B. 243 sq. ft.

C. 273 sq. ft.

D. 303 sq. ft.

Questions 41 and 42 are open-response questions.

- **BE SURE TO ANSWER AND LABEL ALL PARTS OF EACH QUESTION.**
- **Show all your work (diagrams, tables, or computations) in your Student Answer Booklet.**
- **If you do the work in your head, explain in writing how you did the work.**

Write your answer to question 41 in the space provided in your Student Answer Booklet.

41 In a report on the history of irrational numbers, Celine compared three different values that have been used to approximate π. The values are listed below.

$\left(\frac{4}{3}\right)^4$ Egyptian approximation

$\frac{355}{113}$ Chinese approximation

$\frac{22}{7}$ Archimedes' approximation (Greek)

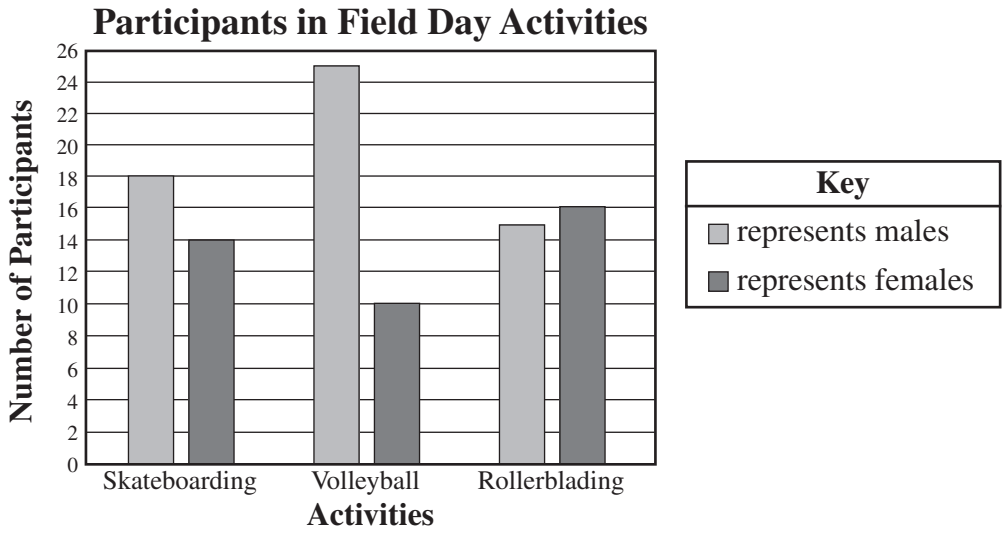
a. Celine compared $\left(\frac{4}{3}\right)^4$, the approximation used by the Egyptians, to $\frac{22}{7}$, a value that she often uses for π. She converted both $\left(\frac{4}{3}\right)^4$ and $\frac{22}{7}$ to decimals rounded to four decimal places (nearest ten-thousandth). To the nearest ten-thousandth, what is the absolute value of the difference between $\left(\frac{4}{3}\right)^4$ and $\frac{22}{7}$? Show or explain how you got your answer.

b. Celine also compared $\frac{355}{113}$, the approximation used by the Chinese, to $\frac{22}{7}$. She converted $\frac{355}{113}$ to a decimal rounded to four decimal places (nearest ten-thousandth). To the nearest ten-thousandth, what is the absolute value of the difference between $\frac{355}{113}$ and $\frac{22}{7}$? Show or explain how you got your answer.

c. Celine knows that $\pi \approx 3.1415927$. Place the four numbers, $\left(\frac{4}{3}\right)^4$, $\frac{355}{113}$, $\frac{22}{7}$, and π in order from **least** to **greatest**. Explain your reasoning.

Write your answer to question 42 in the space provided in your Student Answer Booklet.

42 The double bar graph below shows the number of male and female participants in three different activities at a Field Day. Each person participated in just one activity.



a. Based on the information in the graph, what was the ratio of male to female participants overall? Show or explain how you got your answer.

b. What percent of all of the female participants played volleyball? Show or explain how you got your answer.

c. Sketch and label a circle graph that shows the information given in the graph for the female participants. Your sketch does not have to be exact but should show the sectors relatively proportioned. Explain how you determined the size of each sector.

Massachusetts Comprehensive Assessment System
Grade 10 Mathematics Reference Sheet

AREA FORMULAS

triangle ..... $A = \frac{1}{2}bh$

rectangle ..... $A = bh$

square ..... $A = s^2$

trapezoid ..... $A = \frac{1}{2}h(b_1 + b_2)$

CIRCLE FORMULAS

$C = 2\pi r$

$A = \pi r^2$

VOLUME FORMULAS

cube ..... $V = s^3$
(s = length of an edge)

rectangular prism ..... $V = lwh$

OR

$V = Bh$
(B = area of the base)

sphere ..... $V = \frac{4}{3}\pi r^3$

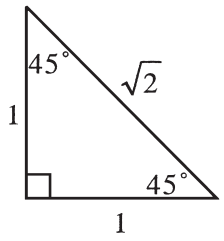
right circular cylinder ..... $V = \pi r^2 h$

right circular cone ..... $V = \frac{1}{3}\pi r^2 h$

right square pyramid ..... $V = \frac{1}{3}s^2 h$

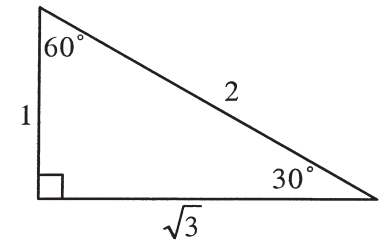
LATERAL SURFACE AREA FORMULAS

rectangular prism ..... $LA = 2(hw) + 2(lh)$
right circular cylinder ..... $LA = 2\pi rh$
right circular cone ..... $LA = \pi r\ell$
right square pyramid ..... $LA = 2s\ell$
(ℓ = slant height)



TOTAL SURFACE AREA FORMULAS

cube ..... $SA = 6s^2$
rectangular prism ..... $SA = 2(lw) + 2(hw) + 2(lh)$
sphere ..... $SA = 4\pi r^2$
right circular cylinder ..... $SA = 2\pi r^2 + 2\pi rh$
right circular cone ..... $SA = \pi r^2 + \pi r\ell$
right square pyramid ..... $SA = s^2 + 2s\ell$
(ℓ = slant height)



Grade 10 Mathematics
Spring 2005 Released Items:
Reporting Categories, Standards, and Correct Answers

Item No.	Page No.	Reporting Category	Standard	Correct Answer (MC/SA)*
1	204	*Number Sense and Operations*	10.N.2	B
2	204	*Data Analysis, Statistics, and Probability*	10.D.1	C
3	205	*Patterns, Relations, and Algebra*	10.P.7	B
4	205	*Number Sense and Operations*	10.N.2	B
5	205	*Patterns, Relations, and Algebra*	10.P.4	C
6	206	*Number Sense and Operations*	10.N.3	B
7	206	*Number Sense and Operations*	10.N.1	A
8	206	*Number Sense and Operations*	10.N.4	C
9	207	*Measurement*	10.M.1	A
10	207	*Patterns, Relations, and Algebra*	10.P.2	A
11	208	*Number Sense and Operations*	10.N.4	C
12	208	*Number Sense and Operations*	10.N.3	C
13	208	*Patterns, Relations, and Algebra*	10.P.3	D
14	209	*Data Analysis, Statistics, and Probability*	10.D.1	C
15	210	*Measurement*	10.M.1	8 inches
16	210	*Number Sense and Operations*	8.N.10	9
17	211	*Patterns, Relations, and Algebra*	10.P.7	
18	212	*Data Analysis, Statistics, and Probability*	10.D.1	any fraction, decimal, or percent approximately equal to $\frac{1}{6}$
19	212	*Geometry*	10.G.3	80°
20	213	*Geometry*	10.G.7	
21	214	*Measurement*	10.M.1	
22	215	*Data Analysis, Statistics, and Probability*	10.D.1	C
23	216	*Data Analysis, Statistics, and Probability*	10.D.1	D
24	216	*Measurement*	10.M.1	A
25	217	*Geometry*	10.G.7	C
26	217	*Patterns, Relations, and Algebra*	10.P.7	A
27	218	*Measurement*	10.M.2	D
28	218	*Data Analysis, Statistics, and Probability*	10.D.1	B
29	219	*Geometry*	10.G.6	B
30	220	*Data Analysis, Statistics, and Probability*	10.D.1	C
31	221	*Patterns, Relations, and Algebra*	10.P.8	
32	222	*Geometry*	10.G.5	C
33	222	*Patterns, Relations, and Algebra*	10.P.6	B
34	222	*Patterns, Relations, and Algebra*	10.P.3	D
35	223	*Data Analysis, Statistics, and Probability*	10.D.2	A
36	223	*Data Analysis, Statistics, and Probability*	10.D.1	C
37	224	*Data Analysis, Statistics, and Probability*	10.D.1	B
38	224	*Geometry*	10.G.5	A
39	225	*Patterns, Relations, and Algebra*	10.P.7	C
40	225	*Measurement*	10.M.1	B
41	226	*Number Sense and Operations*	10.N.2	
42	227	*Data Analysis, Statistics, and Probability*	10.D.1	

* Answers are provided here for multiple-choice and short-answer items only. Sample responses and scoring guidelines for open-response items, which are indicated by shaded cells, will be posted to the Department's Web site later this year.